Application of Vector Autoregression (Var) On Modelling and Forecasting Average Monthly Rainfall and Temperature

Yahaya Musa

Department of Statistics, Faculty of Natural and Applied Sciences

Nasarawa State University Keffi

Ibrahim Ahmad and

Department of Statistics, Faculty of Natural and Applied Sciences

Nasarawa State University Keffi

Bilkisu Maijamaa

Department of Statistics, Faculty of Natural and Applied Sciences

Nasarawa State University Keffi

Email: Maijamaab@Nsuk.Edu.Ng

Abstract

Rainfall and temperature have become the two most natural factor that determines the standard of agricultural production. Sensitivity in climate variability over a long period of time need to be recorded, looking at difference in temporal and spatial scale. The need to understand the nature of the differences in the climate system and their impact on the society and environment is of great interest. This paper tends to apply Vector auto-regressive on modelling and forecasting average monthly rainfall and temperature in Nigeria. A monthly data sourced from World Bank climate portal, from January 1986 to December 2021. Augmented Dickey-Fuller (ADF) a test used to test for stationarity of the trends. Also, the criterion, Alkaike information criterion (AIC) is considered in the model lag selection and the VAR model favored VAR at lag 8. Ordinary least square has been used to estimate the VAR model parameter. Granger causality shows a bi-lateral causation from the temperature during rainfall and from rainfall during different temperature. “Impulse Response Functions” (IRF) and “Forecast Error Variance Decomposition”

(FEVD) were further carried out as a structural analysis between the two variables, it revealed that, rainfall and temperature are interrelated.

Keywords: Rainfall; Forecast Error Variance Decomposition, Temperature; Vector Autoregressive, Modelling, Impulse Response Functions.

1. INTRODUCTION

Rainfall and temperature have become the two most natural factor that determines the standard of agricultural production. The change in rainfall and temperature pattern are especially important to the economy of the country. Nigeria, as a country that mostly depend on agriculture, hence, information on rainfall and temperature probabilities is vital towards determining the best plants and species that are adaptive, the best time for planting or seeding are different. Climate and temperature changes are evident across the globe with extreme weather conditions; this climate change is linked to global warming of the atmosphere which is a growing concern since many decades.

Information about the future rainfall and temperature play important role in making informed decisions. Farmers and other agribusiness managers need forecast about future rainfall and temperature to make investment decisions. Hence, there is need to understand the rainfall and temperature pattern and forecast their future and examine how they relate in Nigeria.

Zhao, J., et al 2019 study on simulating and looking at the relationship between the runoff and some factors such as terraces, grazing fencing, grassing, afforestation, check dams' construction. The model of vector auto-regression was used, and the result has shown lagged precipitation with the implementation of measures of conservation of soil and water.

Adenomon et al., (2013) examined the dynamic relationship between rainfall and temperature time series data in Niger State. VAR model favored VAR at lag 8, which showed relationship between rainfall and temperature. The "Impulse Response Functions" (IRF) and the "Forecast Error Variance Decomposition" depicted that, modelling rainfall and temperature together lead to the improvement of the forecast of rainfall and temperature. Adenomon and Oyejola (2014) forecast meteorological time series data with a reduced for Vector Autoregressive (VAR) Model and three univariate time series techniques. The data for the study was characterized by a stationary time series data, with a bi-directional causality between the relative humidity and temperature. The outcome of the study reveals a strong correlation negatively between the relative humidity and temperature.

(Nwosu C. A., et al 2023) studying the impact of climate change on international trade in Nigeria, a non-recursive structural vector autoregressive (SVAR) model used for the research. IRF reveals that CO_2 emissions causes total trade to decline from the first year up till the third year, therefore, it can be concluded that climate change affect both total trade and total export in the country from annual data on Total Trade, Exchange-rate, Total Export, Real GDP, Carbon-dioxide emissions, and Population growth-rate. The result shows that CO_2 emissions negatively affects total trade.

Chukwudum and Nadarajah (2022) examined Bi-variate value of Rainfall and Temperature analysis in Nigeria. The bivariate extreme cases of monthly rainfall and temperature observed shows, that these variables exhibit inter-relationship such as dry-cold and wet-cold associations. The study revealed that the compound extreme of dry-cold and wet-cold conditions depicted a zero to weak extreme dependence at changing levels of quantile.

2. MATERIAL AND METHODS

2.1 Model Specification

VAR is a statistical model used to analyze the inter-decencies and dynamic relationship between multiple time series variables.

statistically, VAR equations for the two variables as given below.

Rainfall equation

$R_t = c_r + A_{\{rr,1\}} \ R_{\{t-1\}} + A_{\{rt,1\}} \ T_{\{t-1\}} + e_{\{rt\}}$

Temperature equation

$T_t = c_t + A_{\{tr,1\}} \ R_{\{t-1\}} + A_{\{tt,1\}} \ T_{\{t-1\}} + e_{\{tt\}}$

VAR model can also be represented in matrix form as shown below

$$\begin{bmatrix} R_t \\ T_t \end{bmatrix} = \begin{bmatrix} C_r \\ C_t \end{bmatrix} + \begin{bmatrix} A_{rr,1} & A_{rt,1} \\ A_{tr,1} & A_{tt,1} \end{bmatrix} \begin{bmatrix} R_{t-1} \\ T_{t-1} \end{bmatrix} + \begin{bmatrix} \ell_{rt} \\ \ell_{tt} \end{bmatrix}$$

Where:

R_t represent the rainfall variable at time t.

T_t represent the temperature variable at time t.

c_r *and* c_t are the constant terms in the respective equations

$A_{\{rr,1\}}$ *and* $A_{\{rt,1\}}$ are the coefficients of the lagged rainfall (R) and lagged temperature (T) terms in the rainfall equation.

$A_{\{tr,1\}}$ *and* $A_{\{tt,1\}}$ are the coefficients of the lagged rainfall (R) and lagged temperature (T) terms in the temperature equation.

$e_{\{rt\}}$ *and* $e_{\{tt\}}$ are the error terms in the respective equations

2.2 Unit Root Test

"The Augmented Dickey-Fuller" (ADF) test is use to test for the unit root. Suppose we have a series y_t for testing unit root, then ADF model is given as

$$\Delta y_t = \mu + at + \delta y_{t-1} + \sum_{i=1}^{n} \beta \Delta y_{t-1} + e_t$$

Where:

Δy_t is the first difference of y_t *i.e* $y_t - y_{t-1}$

t is the time or trend variable

e_t is a white noise

n is the maximum lag length

μ, δ and β are the parameter to be estimated

The stationarity test hypothesis is as follows:

H_o: $\delta = 0$ "Unit Root" (non-stationarity)

H_a: $\delta \neq 0$ "There is no Unit Root" (stationarity)

If we accept the null hypothesis, we assume that there is a unit root and difference the data before running a regression. If the null hypothesis is rejected, the data are stationary and can be used without differencing

(Dominick and Derrick, 2002)

2.3 Lag length Selection in Vector Autoregressive Models

The Akaike Information Criteria (AIC) is employed in obtaining the optimal lag length (p) for this study. The statistical form of AIC is as follows:

$$\boldsymbol{AIC_{(P)} = In\left|\Sigma_{(P)}\right| + \frac{In\,T}{T}\,pk^2}$$

Where:

Σ = “estimated covariance matrix”

T= “number of observations”

P = “optimal lag length of the VAR Model”

K = “Number of the autoregressive parameters estimated in a VAR(p) model”

2.4 Parameter Estimation

A VAR (p) model can be estimated by the “Least Square”, or “Maximum Likelihood method” or “Bayesian method”. However, Ordinary least square method of had been employed in this study. The Ordinary Least Square (OLS) Method is given as:

$$Y_{t\,=BZ+U}$$

Where:

Y_t = dependent variable in time t (Rainfall and Temperature)

B = the coefficients of the lagged values (A_1, A_2,………A_P)

Z = the lagged values (Y_{t-1},………Y_{t-p+1},)

U = random error (u_1, u_2,…….u_t)

2.5 Diagnostic Check

Model checking or diagnostic check or residual analysis, plays an important role in building models, to ensure an adequate fitted model and to suggest directions for further improvements if needed.

The following will be used to check the model.

(i) Residual plot

(ii) Serial correlation

2.6 Forecasting

The one-step ahead forecasts are represented as below.

$$\hat{y}_{t+1} = \hat{A}_1\, y_t + \hat{A}_2\, y_{t-1} + \cdots + \hat{A}_P\, y_{t-p+1}$$

And successively, we will find $\hat{y}_{t+2}, \hat{y}_{t+3}, \ldots\ldots\ldots \hat{y}_{t+h}, \ldots\ldots\ldots$

Furthermore, let h be the forecast origin, $\ell > 0$ be the forecast horizon and F_h be the information available at h, the ℓ-step ahead minimum mean squared error (MSE) forecast of $z_{h+\ell}$ is given as

$$\hat{Z}_h(\ell) = \hat{\phi}_0 + \sum_{i=1}^{p} \hat{\phi}_i\ \hat{Z}_h(\ell - \mathrm{i})$$

2.7 Granger Causality Test

Granger causality test is a technique for determining whether one-time series is useful in forecasting another (Granger, 1969). A time series X_t is said to be Granger-cause Y_t if it can be shown, usually through a series of F-test on lagged value of X_t (and with lagged value of Y_t also known), that value of X_t provide statistically significant information about future value of Y_t (Gelper and Croux, 2007).

The G ranger causality test is based on the following regression.

$$X_t = \sum_{i=1}^{n} \alpha_i\, Y_{t-i} + \sum_{j=1}^{n} \beta_j\, X_{t-i} + u_{1t}$$

$$Y_t = \sum_{i=1}^{n} \lambda_i\, Y_{t-i} + \sum_{j=1}^{n} \delta_j\, X_{t-j} + u_{2t}$$

Where:

X_t and Y_t represent and the rainfall and temperature variable at time t.

X_{t-j} and Y_{t-i} are the lagged of Rainfall and temperature respectively

α_i and β_j are the regression coefficient for rainfall model

λ_i and δ_j are the regression coefficient for temperature model

u_{1t} and u_{2t} represent the disturbance for the two variables. It is assumed that the disturbance u_{1t} and u_{2t} are uncorrelated.

2.8 IRF

"The Impulse Response Function" (IRF) is used to find out how each endogenous variable responds over time to a shock in its own value and in every other variable.

$$\boldsymbol{Y_t} = \eta + \boldsymbol{\theta_0}\eta_{\boldsymbol{t}} + \boldsymbol{\theta_1}\eta_{\boldsymbol{t-1}} + \boldsymbol{\theta_2}\eta_{\boldsymbol{t-2}}$$

Where:

$\boldsymbol{\theta_0} = \boldsymbol{B^{-1}}$ is a "lower triangular matrix."

$\eta_{\boldsymbol{jt}}$ are the "impulse response to orthogonal shocks "

From this equation we can observe changes in Y_t given a change in the residual.

$\frac{\partial y_{i,}t+s}{\partial n_{i,}t} = \frac{\partial y_{i,}t}{\partial n_{i,}t-s} = \theta^s_{i,j}$ i, j=1,2,…….., n, s > 0

Where

$\theta^s_{i,j}$ is the (i, j)th elements of $\boldsymbol{\theta_s}$

2.9 Forecast Error Variance Decompositions

"Forecast Error Variance Decomposition" (FEVDs) quantify the equation by showing how much of the forecast error variance is caused by the structural shock. it provides a way of accessing the impacts of changes in the variables on each other.

For any given shock ($\eta_{\boldsymbol{t}}$), the "h-step forward forecast error vector" is given by

$$Y_{T+h} - Y_{T+h/T} = \sum_{s=0}^{h-1} \Theta_s \, \eta_{T+h-s}$$

Where $Y_{T+h/T}$ is "h-step forecasts based on information available at time T"

However, for a particular variable $Y_{i+h/T}$ this forecast error has the form.

$Y_{i,T+h} - Y_{i,T+h/T} = \sum_{s=0}^{h-1} \theta_{i1}^{s} \ \eta_{1,T+h-s} \ + \ldots + \sum_{s=0}^{h-1} \theta_{in}^{s} \ \eta_{n,T+h-s}$

Where $\boldsymbol{\sigma}_{\eta j}^{2} = var(\eta_{jt})$ is the function of var($Y_{i,T+h} - Y_{i,T+h/T}$) cause by the shock η_j

Hence the forecast error variance decomposition is given as

$$FEVD_{I,J}\,(h) = \frac{\sigma_{\eta j}^{2} \sum_{s=0}^{h-1} (\boldsymbol{\theta}_{ij}^{s})}{\sigma_{\eta 1}^{2} \sum_{s=0}^{h-1} (\boldsymbol{\theta}_{i1}^{s}) + \cdots + \sigma_{\eta n}^{2} \sum_{s=0}^{h-1} (\boldsymbol{\theta}_{in}^{s})}, i, j = 1, \ldots n$$

3. DISCUSSION AND RESULTS

3.1 Time Series plot

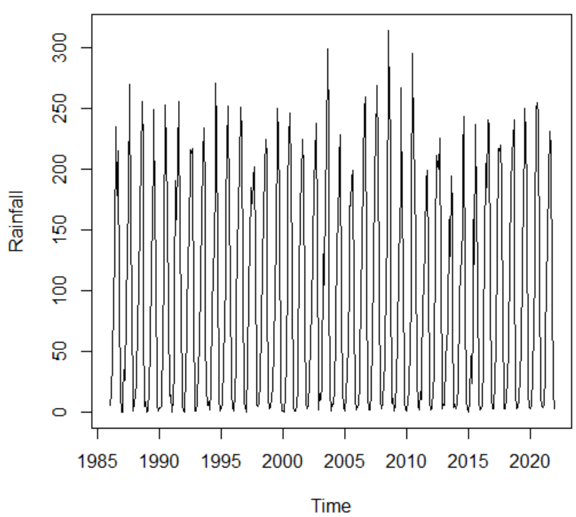


Fig 1: - Time Plot of Average Monthly Rainfall (AMRF)

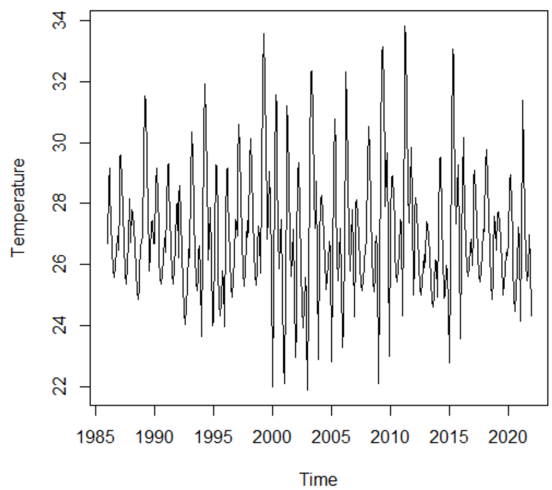


Fig 2: Time Plot of Average Monthly Temperature (AMTEMP)

COMMENT: From both the plots both-variables (Rainfall and Temperature) appear to be stationary over time. A stationarity test is further conducted to back up this claim.

4. EMPIRICAL RESULTS

Table:1 Descriptive Statistics

	N	Mean	Std. Deviation	Variance	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Rainfall	432	95.3032	87.09162	7584.950	.474	.117	-1.169	.234
Temperature	432	27.0968	2.05798	4.235	.622	.117	.713	.234
Valid N (listwise)	432							

COMMENT: The descriptive statistics on rainfall and temperature are presented in table 1.

4.1 Stationarity Test

Table 2: ADF Unit Root Test for stationarity for Rainfall variable

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-16.59248	0.0000
Test critical values:	1% level	-3.445590	
	5% level	-2.868153	
	10% level	-2.570357	

Table 3: ADF Unit Root Test for stationarity for Temperature variable

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.743157	0.0000
Test critical values:	1% level	-3.445590	
	5% level	-2.868153	
	10% level	-2.570357	

COMMENTS: The ADF test for rainfall and temperature series are presented in able 2 and table 3. The test was carried out without trend.

4.1.1 The VAR Model and Lag Selection

Table 4: Lag length selection

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-3396.825	NA	31458.92	16.03219	16.05129	16.03974
1	-3008.503	771.1487	5133.788	14.21935	14.27666	14.24199
2	-2796.131	419.7351	1921.217	13.23647	13.33198	13.27420
3	-2722.143	145.5326	1381.033	12.90634	13.04005	12.95917
4	-2695.110	52.91854	1238.860	12.79769	12.96961	12.86561
5	-2654.463	79.18415	1042.207	12.62483	12.83495	12.70785
6	-2629.010	49.34589	941.9163	12.52363	12.77196	12.62175
7	-2585.392	84.14934*	781.3794	12.33676	12.62329*	12.44997*
8	-2580.937	8.552592	779.7255*	12.33461*	12.65935	12.46291

COMMENTS: VAR models for a given lag was considered, and lag 8 is chosen by AIC criterion as the optimal lag for the analysis.

Table 5: VAR Model Estimation at lag 8

Variable	Rainfall	Temperature
RAINFALL (-8)	-0.112834 (0.05015) [-2.25011]	0.000875 (0.00161) [0.54247]
TEMPERATURE (-8)	1.646948 (1.52387) [1.08076]	1.646948 (1.52387) [1.08076]
C	273.2897 (48.6741) [5.61469]	273.2897 (48.6741) [5.61469]

COMMENTS: Table 5 shows the output of the vector autoregression (VAR) model estimation at lag 8. From the result, rainfall is not significant enough to predict itself that is, it does not have a significance influence by itself with t-statistics of (-2.2501) while temperature strongly predict rainfall with t-statistics of (1.0807). hence it can be said that, the percentage increase on temperature account for 164% increase in rainfall on average. the pass realization of rainfall is associated with 0.008% increase in temperature on average, while the percentage increase on temperature account for 164% increase in itself on average.

4.2 VAR Residual Diagnostic

Table 6: Test for Residual Serial Correlation

Null hypothesis: No serial correlation at lag h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	11.33965	4	0.0230	2.851849	(4, 784.0)	0.0230
2	27.30420	4	0.0000	6.937302	(4, 784.0)	0.0000
3	9.836400	4	0.0433	2.471419	(4, 784.0)	0.0433
4	19.84368	4	0.0005	5.017753	(4, 784.0)	0.0005
5	3.845297	4	0.4273	0.962455	(4, 784.0)	0.4273
6	7.431009	4	0.1148	1.864195	(4, 784.0)	0.1148
7	11.89595	4	0.0181	2.992818	(4, 784.0)	0.0181
8	2.231919	4	0.6932	0.558062	(4, 784.0)	0.6932

COMMENTS: From Table 6, at 5% significance level, it clearly shows that, using 8 lag is appropriate since the p-value is greater than alpha at lag 8, it means there is no serial correlation at lag 8 and is long enough to capture monthly data dynamics.

Table 7: "VAR Residual Normality Test"

Component	Skewness	Chi-sq	df	Prob.*
1	-0.235785	0.259440	1	0.6105
2	0.784786	2.874149	1	0.0900
Joint		3.133589	2	0.2087

Component	Kurtosis	Chi-sq	df	Prob.
1	2.551644	0.234527	1	0.6282
2	3.851243	0.845383	1	0.3579
Joint		1.079910	2	0.5828

Component	Jarque-Bera	df	Prob.
1	0.493967	2	0.7812
2	3.719532	2	0.1557
Joint	4.213499	4	0.3779

COMMENTS: From table 7, the residual of both rainfall and temperature are normally distributed. The joint normality test for the whole variable also show that the residual is normally distributed at lag 8.

4.2.1 The "Granger" causality test

Table 8: Granger Causality Test

Dependent variable: RAINFALL

Excluded	Chi-sq	df	Prob.
TEMPERATURE	94.30607	8	0.0000
All	94.30607	8	0.0000

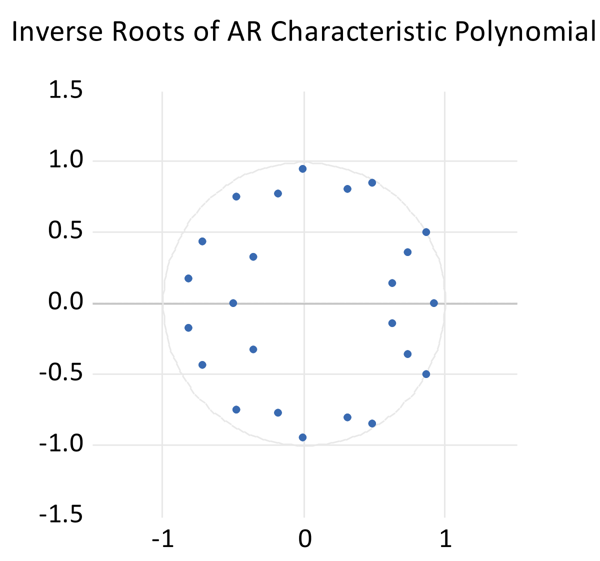
Dependent variable: TEMPERATURE

Excluded	Chi-sq	df	Prob.
RAINFALL	136.3028	8	0.0000
All	136.3028	8	0.0000

COMMENTS: The Granger causality test shows a bi-lateral relationship between Rainfall and Temperature as seen in the table 8.

4.2.2 Stability Condition of VAR model

Figure 3: "Stability Condition of VAR" Model (unit root circle)



COMMENTS: This is the AR root graph that shows the unit circle. It clearly shows that all the root lies inside the circle. This shows that, the VAR model satisfies stability condition. However, for more clarity, the AR root table is shown below.

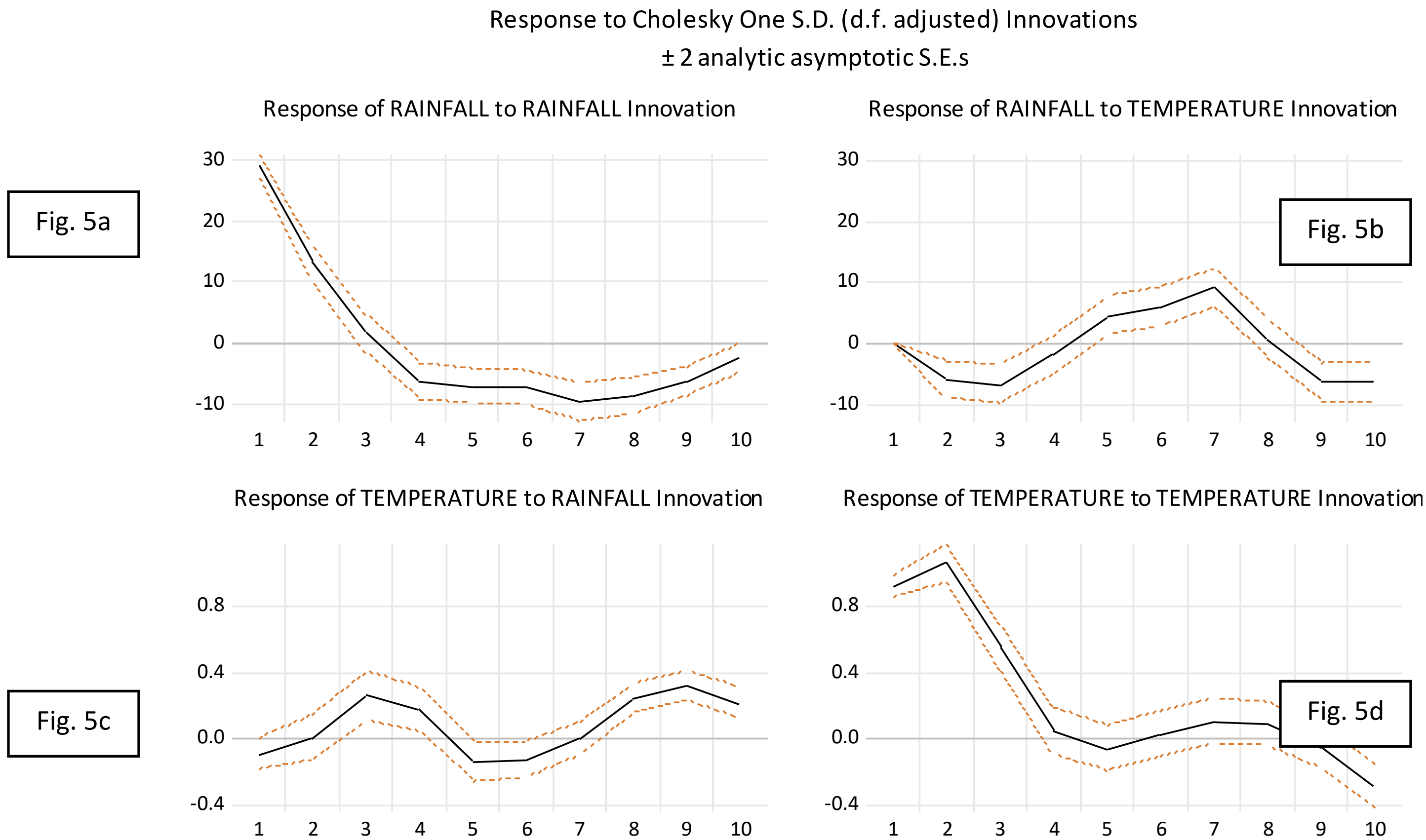
Table 9: Eigenvalue stability condition

Root	Modulus
0.864973 + 0.499199i	0.998688
0.864973 - 0.499199i	0.998688
0.492246 + 0.851357i	0.983420
0.492246 - 0.851357i	0.983420
-0.005936 + 0.954351i	0.954370
-0.005936 - 0.954351i	0.954370
0.924122	0.924122
-0.480396 - 0.756436i	0.896089
-0.480396 + 0.756436i	0.896089
0.316485 - 0.810057i	0.869687
0.316485 + 0.810057i	0.869687
-0.716140 - 0.440578i	0.840812
-0.716140 + 0.440578i	0.840812
-0.810772 + 0.173892i	0.829211
-0.810772 - 0.173892i	0.829211
0.742787 - 0.361101i	0.825909
0.742787 + 0.361101i	0.825909
-0.181460 - 0.777259i	0.798160
-0.181460 + 0.777259i	0.798160
0.632523 - 0.146318i	0.649226
0.632523 + 0.146318i	0.649226
-0.496846	0.496846
-0.354862 + 0.325168i	0.481312
-0.354862 - 0.325168i	0.481312

COMMENTS: Table 9, show the results that revealed that all the inverse roots of the characteristic AR polynomial have modulus less than 1. Meaning that, the estimate VAR is stable, and both the variables are stationary.

4.3 Impulse Response Functions

Fig 5: The IRF for rainfall and Temperature time series data



COMMENTS: The main diagonal graph shows the effect of a shock of a variable to itself (shock of Rainfall on Rainfall and a shock of Temperature on Temperature). However, we are more interested in seeing the cross graph (the response of rainfall to a shock on temperature and response of temperature to a shock on rainfall).

From fig.5b, when temperature increase in one standard deviation shock, the rainfall starts decreasing after the first period, reaching a maximum percentage variation of -8 at around the third month then starts increasing again (temperature has a gradual negative effect on rainfall)

From fig.5c, when rainfall increase in one standard deviation shock, the temperature starts increasing after the first period, reaching a maximum percentage variation of -0.3 at around the third month then starts decreasing again (rainfall has a gradual positive effect on temperature)

4.4 Variance Decompositions

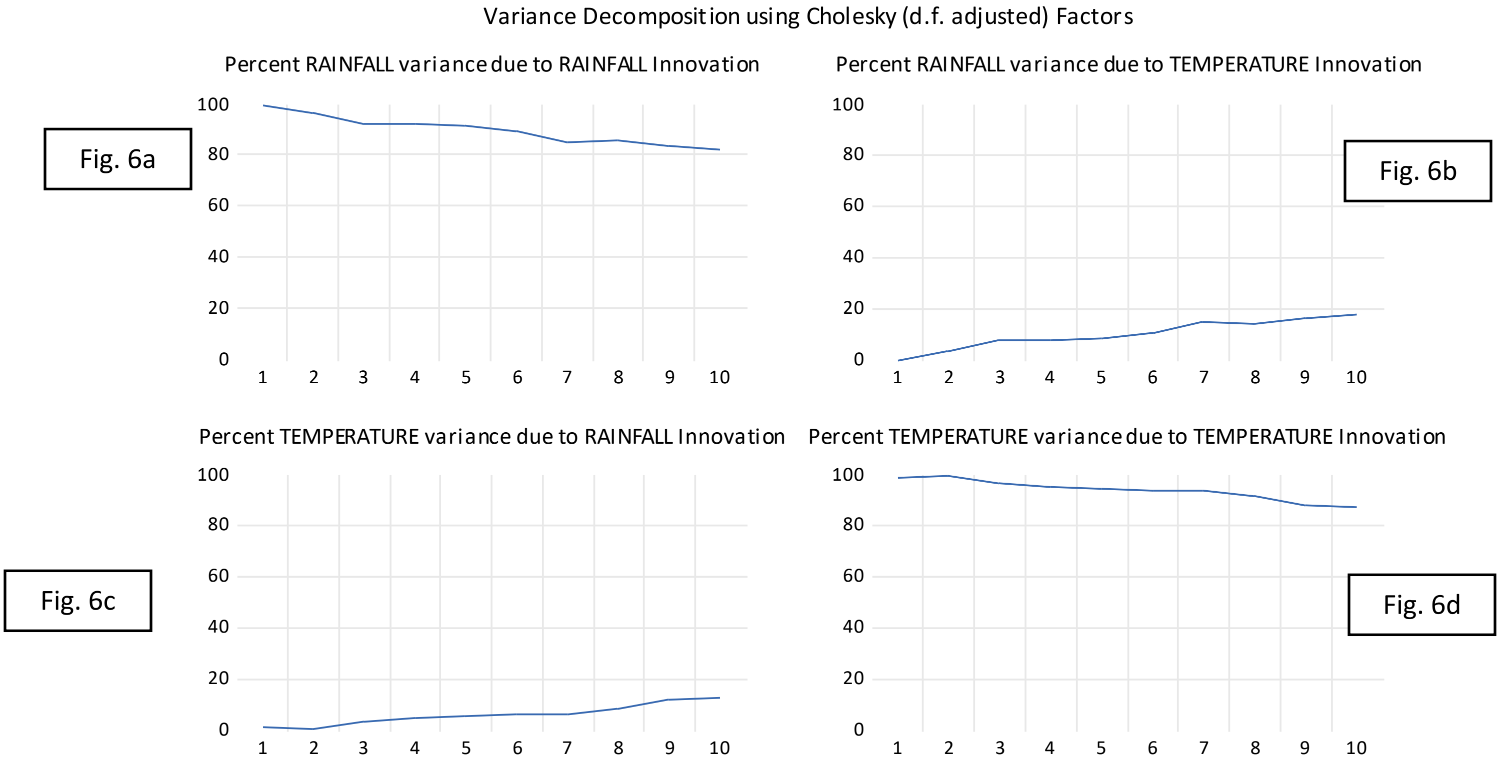
Table 10: "Variance Decomposition"

Period	S.E.	RAINFALL	TEMPERATURE
Variance Decomposition of RAINFALL:			
1	28.96819	100.0000	0.000000
2	32.41759	96.65226	3.347740
3	33.15157	92.70452	7.295483
4	33.79118	92.67023	7.329767
5	34.81667	91.39605	8.603952
6	36.03215	89.33310	10.66690
7	38.36998	84.98928	15.01072
8	39.29468	85.65689	14.34311
9	40.27286	83.91813	16.08187
10	40.81545	82.03043	17.96957
Variance Decomposition of TEMPERATURE:			
Period	S.E.	RAINFALL	TEMPERATURE
1	0.931367	0.982573	99.01743
2	1.414317	0.428333	99.57167
3	1.540729	3.219746	96.78025
4	1.551545	4.488420	95.51158
5	1.558957	5.245486	94.75451
6	1.564531	5.885737	94.11426
7	1.567793	5.861614	94.13839
8	1.589011	8.022012	91.97799
9	1.622040	11.60211	88.39789
10	1.662728	12.70659	87.29341

COMMENTS: Temperature have no contemporaneous effect on rainfall at the first period, however, the temperature explains 3% of the variation in rainfall at the second period. The result simply shows that as the periods increases over time, change in rainfall have to do with the temperature.

Rainfall has a contemporaneous effect of 0.9% on temperature at the first period, however, the result also shows that as the periods increases over time, change in rainfall have to do with the temperature.

Fig 6: Decomposition of Variance from VAR



COMMENTS: The Variance decomposition in Fig 6a and Fig 6d have the same manner of appearance, same goes to Fig 6b and Fig 6c. This shows that, some percentage of variance in rainfall is explained by innovation in rainfall, and some were explained by innovation in temperature. Also, some percentage of variance in temperature is explained by innovation in temperature, and some were explained by innovation in rainfall.

5. CONCLUSION

Two climate variables were examined in this study: rainfall and temperature. Bi-lateral causality dictated among these variables under Granger causality, and it shows that the variables are interrelated. From the model, rainfall does not have a significance influence by itself with t-statistics of (-2.2501) while temperature strongly predict rainfall with t-statistics of (1.0807). Impulse Response function and forecast error variance decomposition was utilized, it shows that, when temperature increase in one standard deviation shock, the rainfall starts decreasing on a short run, and increase on a long run, on the other hand, when rainfall increase in one standard deviation shock, the temperature starts increasing after a short run, and decrease on a log run. temperature explains 3% of the variation in rainfall after a short run and keep on increasing at the long run, Rainfall explain 0.9% of the variation temperature after a short run and keep increasing after a long run.

REFERENCES

6. Abaje I., B. and Oladipo E., O. (2019): observation of recent change in the temperature and Rainfall Conditions in Kaduna State, Nigeria. Ghana Journal of Geography Volume 11(2), 2019 pages 127-157.
7. Abaje I.B., Pattience J.A., Mansur B.M 2018) Spatio-temporal analysis of Rainfall distribution in Kaduna state Nigeria. Ghana Journal of Geography 10(1), 1-12, 2018
8. Abasi R.A., Ikubuwaje C.O. (2012) Analytical study of rainfall and temperature trend in catchment states and stations of the Benin-Owena River basin, Nigeria. Journal of Environment and Earth Sciences 2(3), 9-12, 2012
9. Abbate, P.E., Dardanelli, J. L., Cantarero, G., Maturano, M., Melchiori, R.J.M and Suero, E. E (2004) "Climate and water availability effects on water-use efficiency in wheat," Crop Science, 44(2):474-483
10. Adenomon M. O., and Oyejola A. B. (2014): Forecasting Meteorological Time series data with a reduced form Vector Autoregressive (VAR) Model and three univariate time series techniques. Social and Basic Sciences Resaerch Review, 2(3): 139-152.
11. Adenomon, M. O. and Okperie, P. E. (2013): Improving Meteorological Time Series data forecast with reduced form Vector Autoregressive (VAR) Model. A paper presented at 9th Annual National Conference at the school of Applied Arts and Sciences. The Federal Polytechnic, Bida.
12. Adenomon, M. O., Ojehomon, V.E.T. and Oyejola, B. A. (2013): Modelling the D y n a m i c Relationship between Rainfall and Temperature Time Series data in Niger State, Nigeria: Mthematical Theory and Modelling. 3(4):53-70

13. Akinsanola A., A. and Ogunjobi K., O. (2014): Investigation of Rainfall and Temperature Variabilities in Nigeria. Double Blind Peer Reviewed Internatinal Research Journal. Volume 14, No.3, Version 1.0, Year 2014.
14. Akinsola, A.A. and Ogunjobi, K.O. (2014). Analysis of Rainfall and Temperature variability over Nigeria. Global Journal of Human Social Science: B Geography, Geo-Sciences, Environmental Disaster Management, 14(3). In Press.
15. Ansari, M. I. and Ahmed, S. M (2007): Does money matter? Evidence from vector Error-correction for mexico. The Journal of developing Areas. 14(1):185-202.
16. Arpita P. and Netrananda S. (2019): Trend analysis of seasonal rainfall and temperature pattern in Kalahandi. Atmospheric Science Letter/ volume 20.
    A. S.M. Mohiul, Md. Habibur R., Farjana S., Muhammad S.H. (2022). Analysis of Spatially Distributed Monthly Rainfall and Temperature over Bangladesh. International Journal of Enviroment and Climate Change Volume 12(issue 12):693-708
17. Beenstock, M. and Felsenstein, D. (2007). Spatial Vector Autoregression: Economic Analysis 2(2):167-196
18. Besler, D. A. (1986): Forecasting Multiple Time Series with prior information. American Journal of Agrcultural economics. 72(3):788-792.
19. Chowdhury, A. R., (1986): Vector Autoregression as alternative Macro-modelling Tchniques: The Banglash Development Studies. 14(2):21-32
20. Chukwudum C. Q., and Nadarajah S. (2022): examining Bivariate value of Rainfall and Temperature Analysis in Nigeria. Enviromental Modelling and Assessment 27, 343-362 (2022).
21. Dau A.U., Muhammad S.K., Ujih O.U., Ismail H. (2021). Rainfall and Temperature Dynamics As Evidence of Climate Variability In Northern Nigeria.
22. David E., Precious E., Bridget E. D., & Susan I.A. (2023). Rainfall and Temperature Variations in a Dry Tropical Environment of Nigeria. Journal of Atmospheric Science Reseach 6(2):50-57
23. Dominick, S. and Derrick, R. (2002). Schaum`s Outline of the Theory and problems of Statistics and Economics (2nd ed). New York: McGraw-Hill Company

24. Egbinola C.N., Amanambu C.A (2013) Climate variation assessment based on rainfall and temperature in Ibadan, South-Western, Nigeria. Journal of Environment and Earth Sciences 3(11), 32-45, 2013
25. Ekwe, M C., Joshua, J. K., Igwe, J. E, and Osinowo, A. A, (2014): Mathematical Study of Monthly and Annual Rainfall Trends in Nasarawa State, Nigeria. IOSR Journal of Mathematics (IOSR-JM); 10(1): pp 56-62
26. Estenson, P.S(1992): The Keynesian Theory of the price level: An econometric Evaluation using Vector Autoregression Model. Journal of post Keynesian Economics 14(4):547-560
27. Granger, C. W.J.(1969). "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods". Econometrica 37(3):424-438
28. Gujarati, D.N. (2003). Basic Econometrics (4th ed). New Delhi: the McGraw-Hill Co.
29. Haden, K. L., and VanTassel, L. W. (1988): Application of Autoregression to Dynamic relationship within the U.S Diary Sector. North Central Journal of Agricultural Economics. 10(2):209-216
30. Hulme, M., Osborn, T.J. and Johns, T.C. (1998) Precipitation sensitivity to global warming: Comparison of observations with HADCM2 simulations. Geophysical Research Letters, 25, 3379-3382.
31. Ibrahim, R. B., Ogundahunsi, D. S., Muili, A. B., Raheem, W. M., Olayode, O., & Adedotun, S. B. (2022) Time Series Forecasting of Climate Variables in Three Selected Major Cities in Nigeria.*Available at SSRN 4429271*.
32. Ikebude, C., Nwaogazie, I. L., & Sam, M. G. (2022). Climate change and trend analysis of 24-hourly annual maximum series using Mann-Kendall and Sen slope methods for rainfall IDF modeling. *International Journal of Environment and Climate Change*, 44-60.
33. IPCC. (2007) Intergovernmental Panel on Climate Change.http//www.ipcc.inf/topic.climate
34. Jiahao T., Seokhyeon K., Ashish S. (2023). How does increasing temperature affect the sub-annual distribution of monthly rainfall. Enviromental Research Climate 2(1)

35. Kamara, S.I. (1986). The origin and types of rainfall in West Africa. Weather, 41: 48-56.
36. Karimuzzaman, M., & Moyazzem Hossain, M. (2020). Forecasting performance of nonlinear time-series models: an application to weather variable. *Modeling Earth Systems and Environment*, *6*(4), 2451-2463.
37. Kayano, M.T. and Sansígolo, C. (2008) Interannual to decadal variations of precipitation and daily maximum and daily minimum temperatures in southern Brazil. Theoretical and Applied Climatology, 97, 81-90. doi:10.1007/s00704-008-0050-4
38. Kaylen, M. S. (1988): Vector Autoregression Forecasting Model.Recent Development Applied to U.S Hog Market. American Journal of Agricultural Economics 70(3):710-712
39. Kazeem, R. A., Amakor, J. U., Ikumapayi, O. M., Afolalu, S. A., & Oke, W. A. (2022). Modelling the Effect of Temperature on Power Generation at a Nigerian Agricultural Institute.*Mathematical Modelling of Engineering Problems*, *9*(3).
40. Lutkepohl, H. and Breitung, J. (1997). Impulse Response Analysis of Vector Autoregressive process: System Dynamic in Economic and financial model.
41. Lutkepohl, H. (2005). New introduction to Multiple Time Series Analysis: New York: Springer Berlin Heidelberg.
42. Mequannt M., Fikadu Y., and Jurgen S. (2021): Time series trend analysis and farmer perceptions of rain fall and temperature in northwestern Ethiopia. Environment, Development and Sustainability. 23, 12904-12924 (2021)
43. Mina M. H., and Sayedul A., (2012): Identifying the dependency pattern of daily rainfall of Dhaka station in Bangladesh using Markov chain and logistic regression model Agricultural Sciences, 3(3): 385-391 http://dx.doi.org/10.4236/
44. Mojolaoluwa D., Emmanuel O. E., and Stephen E. (2017): Analysis of Rainfall and Temperature over Climatic Zones in Nigeria. Journal of Geography, Enviroment and Science International 11(2); 1-14, 2017.

45. Nwosu Chinedu, A., Kenneth, E., & Oyelade Olayinka, O. (2023). Climate Change and International Trade in Nigeria: Structural Vector Autoregression Approach.
46. Oguntude P.G., Temitope A.O., Akinwumiju A.S., Fasinmirin J.T. (2019) Analysis of recente change in Rainfall and Drought Indices in Nigeria. Hydrological Sciences Journal 64(14) 1755-1768, 2019
47. Oyeleke O.,O. (2021). Analysis of decadal Rainfall and Temperature Trend in Warri, Nigeria. DOI:10.24018/ejgeo.2021.2.2.120
48. Park, T. (1990): Forecasting Evaluation for Multivariate Time Series Models. The U.S Cattle Market. Western Journal of Agricultural Economics. 15(1):133-143
49. Salim, R., Hassan, K., & Rahman, S. (2020). Impact of R&D expenditures, rainfall and temperature variations in agricultural productivity: empirical evidence from Bangladesh. *Applied Economics*, *52*(27), 2977-2990.
50. Sims, C. A (1980). “Macro Economics and Reality” Econometrica. 48:1-48
51. Soumik R., Soumitra S. D., Pradeep M., Abdullah M.G.A. (2021) Time Series SARIMA Modelling and Forecasting of Monthly Rainfall and Temperature in the South Asia Countries.
52. Sivajothi R., Kanagasabai K. (2019). Forecasting of Eainfall, Average Temperature, Vapor Pressure and Cloud Cover Using Vector Autoregression Model. Journal of Computational and Theoretical Nanoscience 16(5):1862-1869
53. Verbeek (2017) Exact smoothing for stationary and Non-statationary time series. International Journal for forecasting 16(1), 59-69
54. Verere S.B., Emmanuel E., Beauty E. (2023). Spatiotemporal Trend and Variability Analysis of Rainfall and Temperature over Benin Metropolitan Region, Edo State, Nigeria. Geography Environmenta Sustainability 16(1):6-15
55. WHO. (2019) World Health Organization [Online].http://www.who.inf/topics/material.climate
56. Yusuf N., Daniel O., Ibrahim M., Samson A., Rabia S. (2017) A study of the surface air temperature variations in Nigeria. The Open Atmospheric Science Journal 11(1), 2017

57. Zhao, J., Mu, X., & Gao, P. (2019). Dynamic response of runoff to soil and water conservation measures and precipitation based on VAR model. *Hydrology Research*,*50*(3), 837-848.